1.$\sqrt{(x+1)^2+(y-5)^2}=3\sqrt{(x-2)^2+(y-7)^2}\Rightarrow 8x^2+8y^2-38x-116y+451=0$

2.已知$x^2+y^2+z^2=15^2$且$x+y+z=\dfrac{80}{4}=20$，設表面積爲A

$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)=x^2+y^2+z^2+A$

$20^2=15^2+A\Rightarrow A=175$

3.(1)15 爲斜邊→$11^2+k^2-15^2<0\Rightarrow k^2<102\Rightarrow k<\sqrt{102}$且$k>15-11=4$

$\therefore 4<k<10.\sim$ 有 6 個

(2) k爲斜邊→$11^2+15^2-k^2<0\Rightarrow k^2>346\Rightarrow k>\sqrt{346}$且$k<15+11=26$

$\therefore 18.\sim<k<26$ 有 7 個

共 6+7=13 個

4.令 C($-2+2t$ ，$1-t$)、D($-2-s$ ，$4+2s$)且在$y=x-6$上

$\begin{cases}1-t=(-2+2t)-6\\4+2s=(-2-s)-6\end{cases}\Rightarrow\begin{cases}t=3\\s=-4\end{cases}\Rightarrow\begin{cases}C(4,-2)\\D(2,-4)\end{cases}$ 所求$\sqrt{2^2+2^2}=2\sqrt{2}$

5.$\cos\theta=\dfrac{(\sqrt{5})^2+(\sqrt{13})^2-4^2}{2\cdot\sqrt{5}\cdot\sqrt{13}}=\dfrac{1}{\sqrt{65}}$ $\therefore\sin\theta=\dfrac{8}{\sqrt{65}}$

面積爲$\dfrac{1}{2}\cdot\sqrt{5}\cdot\sqrt{13}\cdot\sin\theta=\dfrac{1}{2}\cdot\sqrt{5}\cdot\sqrt{13}\cdot\dfrac{8}{\sqrt{65}}=4$

6. (A)$a_1=18^2=324$ (B)$a_{17}=2^2=4$ (C)$a_9\div a_{14}=10^2\div 5^2=4$

(D)$a_1+a_2+\cdots+a_{18}=18^2+17^2+\cdots+1^2=\dfrac{18\cdot 19\cdot 37}{6}\neq 18^3$ 故選(D)

7.令$x=21k+2$ →191、212、233、254、275、296 其中只有 233 被 5 除餘 3
故選(B)

8.因爲爲實係數，所以虛根共軛可知有一因式x^2+x+1且另一根爲實根

則$x^3+bx^2+cx+1=(x^2+x+1)(x+1)=0=x^3+2x^2+2x+1$

所求 2+2=4

9. $f(x)=a^x$爲指數函數，且$a>1$，故爲遞增函數，恆正(A)(B)皆對

(C) $f(x)f(y)=a^x\cdot a^y=a^{x+y}=f(x+y)$正確 (D)不一定 ，故選(D)

10.令$\dfrac{x^2-1}{x^2+1}=a^x\Rightarrow x^2=\dfrac{1+a}{1-a}$ $\therefore f(x)=\dfrac{1+x}{1-x}$ ，$f'(x)=\dfrac{(1-x)+(1+x)}{(1-x)^2}=\dfrac{2}{(1-x)^2}$

$f(0)=1$，$f'(0)=2$ ，選(D)

11.$\sqrt{14-4\sqrt{10}}=\sqrt{10}-\sqrt{4}=1.\sim\therefore a=1$ 、$b=\sqrt{10}-3$

$\dfrac{1}{a+b+5}-\dfrac{1}{b}=\dfrac{1}{1+\sqrt{10}-3+5}-\dfrac{1}{\sqrt{10}-3}=(\sqrt{10}-3)-(\sqrt{10}+3)=-6$

12. $\log_6[\sqrt{5}(\sqrt{14-4\sqrt{6}}+\sqrt{5+2\sqrt{6}})]=\log_6[\sqrt{5}(\sqrt{12}-\sqrt{2}+\sqrt{3}+\sqrt{2})]=\log_6 3\sqrt{15}$

$=\dfrac{\log 3\sqrt{15}}{\log 6}=\dfrac{b+\frac{1}{2}(b+1-a)}{a+b}=\dfrac{3b+1-a}{2a+2b}$

13.兩對角線長分別爲 $\sqrt{5^2+8^2-2\cdot5\cdot8\cdot\cos60^\circ}=7$ 、 $\sqrt{5^2+8^2-2\cdot5\cdot8\cdot\cos120^\circ}=\sqrt{129}$

利用平行四邊形面積相等 $5\cdot8\cdot\sin60^\circ=\dfrac{1}{2}\cdot7\cdot\sqrt{129}\cdot\sin\theta\Rightarrow\sin\theta=\dfrac{40\sqrt{3}}{7\sqrt{129}}=\dfrac{40}{7\sqrt{43}}$

14.柯西不等式 $x^2+2y^2\ge\dfrac{(3x+4y)^2}{3^2+(2\sqrt{2})^2}=\dfrac{25}{17}$

15. $2\cdot C_4^{10}=420$

16. $C_0^n+\dfrac{1}{2}C_1^n+\dfrac{1}{4}C_2^n+\cdots+\dfrac{1}{2^n}C_n^n>100\Rightarrow(1+\dfrac{1}{2})^n>100\Rightarrow n>\dfrac{\log100}{\log\frac{3}{2}}=11.\sim$

$\therefore n=12$

17. $40\times1.5\times x-(90\times1.5-100)=59x$ ， $x=35$

18. $\dfrac{\frac{7!}{3!}\times2}{7!}=\dfrac{2}{3!}=\dfrac{1}{3}$

19. $\mathrm{P(A'\cup B')=P(A\cap B)'}=1-P(\mathrm{A\cap B})=1-\dfrac{2}{5}\cdot\dfrac{1}{3}=\dfrac{13}{15}$

20.一局後甲爲紅紅、紅白、白白機率分別爲 $\dfrac{1}{4}$ 、 $\dfrac{1}{2}$ 、 $\dfrac{1}{4}$

所求 $\dfrac{1}{4}\times1+\dfrac{1}{2}\times\dfrac{1}{2}+\dfrac{1}{4}\times1=\dfrac{3}{4}$

21.向量 $AB=(-5,2,1)$ 、向量 $BC=(4,1,-1)$ ，

法向量 $N=(\begin{vmatrix}2&1\\1&-1\end{vmatrix},\begin{vmatrix}1&-5\\-1&4\end{vmatrix},\begin{vmatrix}-5&2\\4&1\end{vmatrix})=(-3,-1,-13)$ ，故選(A)

22. $a+b+c+abc=99$

(1)三數皆爲奇數→和爲偶(矛盾)

(2)二奇一偶→和爲偶(矛盾)

(3)一奇二偶→和爲奇，又偶數之質數只有 2，令三數爲 x 、2、2 $\therefore x+2+2+4x=99$ ， $x=19$

所求 2+2+19=23

23. $\begin{cases}\alpha+\beta=b\\\alpha\beta=3c\end{cases}$ 且 $1<\alpha<2$ ， $5<\beta<6$ ，則 $\begin{cases}6<\alpha+\beta<8\\5<\alpha\beta<12\end{cases}$ $\therefore b=7$ ， $c=2$ 或 3

但若 $c=2$ 則 $x^2-7x+6=0$ 兩根爲 1、6(不合)

所求 7+3=10

24.和為$\dfrac{n(n+1)}{2}$，$\sum_{n=1}^{62} n = 1953 < 2003 < \sum_{n=1}^{63} 63 = 2016$，該數為 2016－2003=13

25. $(10a+b)^2 = 100a^2 + 20ab + b^2$，十位數字是否為奇數受$b$影響

當b=4、6 時，十位數字為奇數，共有$10\times 2 = 20$個(a可為 0~9)

26. $f(x) = a(x+1)^2 + b$,可知對稱軸為$x = -1$

$\begin{cases} f(-4) = 9a + b > 0 \\ f(-5) = 16a + b < 0 \end{cases} \Rightarrow a < 0, b > 0$ 有極大值

$b = f(-1) > f(0) > f(2) = f(-4) > 0 \Rightarrow b > f(0) > f(2)$

27. $29x + 145y = 29(x+5y)$為完全立方數，$x + 5y = 29^2 = 841$

x、$y \in N$，則$x+y$最小值 1+168=169

28.令分別為x、y、z張

$\begin{cases} x + y + z = 24 \\ 100x + 200y + 500z = 10000 \end{cases} \Rightarrow \begin{cases} x + y + z = 24 \\ x + 2y + 5z = 100 \end{cases} \Rightarrow x = 2, y = 4, z = 18$

29. $\begin{cases} a + b < c + d \ldots(1) \\ b + c < d + e \ldots(2) \\ c + d < e + a \ldots(3) \\ d + e < a + b \ldots(4) \end{cases} \Rightarrow \begin{cases} (1)+(4) \to e < c \\ (1)+(3) \to b < e \\ (2)+(4) \to c < a \end{cases} \Rightarrow b < e < c < a$,由(4)知$d < a$

30.(1)令$\begin{cases} a = 105d \\ b = d \end{cases}$且$a - b = 120 \Rightarrow 105d = 120 + d, d = \dfrac{120}{104} = \dfrac{15}{13}$(不合)

(2)$\begin{cases} a = 35d \\ b = 3d \end{cases}$且$a - b = 120 \Rightarrow 35d = 120 + 3d, d = \dfrac{120}{32} = \dfrac{15}{4}$(不合)

(3)$\begin{cases} a = 21d \\ b = 5d \end{cases}$且$a - b = 120 \Rightarrow 21d = 120 + 5d, d = \dfrac{120}{16} = \dfrac{15}{2}$(不合)

(4)$\begin{cases} a = 15d \\ b = 7d \end{cases}$且$a - b = 120 \Rightarrow 15d = 120 + 7d, d = \dfrac{120}{8} = 15$

$\therefore a = 15^2 = 225$

31.令$\angle ACI = x^\circ$，則$\angle ACB = \angle ABC = 2x^\circ$、$\angle IAC = \dfrac{180^\circ - 4x^\circ}{2} = 90^\circ - 2x^\circ$

$180^\circ = 126^\circ + 90^\circ - 2x^\circ + x^\circ \Rightarrow x = 36^\circ \Rightarrow \angle CAB = 180^\circ - 2\times 2\times 36^\circ = 36^\circ$

32. $\left(1+\dfrac{1}{1\times 3}\right)\left(1+\dfrac{1}{2\times 4}\right)\left(1+\dfrac{1}{3\times 5}\right)\cdots\left(1+\dfrac{1}{93\times 95}\right)\left(1+\dfrac{1}{94\times 96}\right)$

$= \dfrac{2\times 2}{1\times 3}\cdot\dfrac{3\times 3}{2\times 4}\cdot\dfrac{4\times 4}{3\times 5}\cdot\cdots\cdot\dfrac{94\times 94}{93\times 95}\cdot\dfrac{95\times 95}{94\times 96} = \dfrac{2}{1}\cdot\dfrac{95}{96} = \dfrac{95}{48}$

33. $\dfrac{930\times(\frac{1}{93}+10)}{2} - 93\times(0+1+2+\cdots+9) - 10 = 460$

34. $\angle A = 180^\circ - \dfrac{360^\circ - 23^\circ - 33^\circ}{2} = 28^\circ$

35. $\lim_{x\to 0^+}(\frac{1}{x})^x = \lim_{t\to+\infty} t^{\frac{1}{t}} = \lim_{t\to+\infty} e^{\frac{\ln t}{t}} = e^0 = 1$

36. $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} \overset{令 y=mx}{=} \lim_{x\to 0}\frac{x^2-(mx)^2}{x^2+(mx)^2} = \frac{1-m^2}{1+m^2}$ →不存在

37. $x^2 = 2x+3 \Rightarrow x = 3,-1 \therefore \int_{-1}^{3} 2x+3-x^2 dx = \left[x^2+3x-\frac{1}{3}x^3\right]_{-1}^{3} = \frac{32}{3}$

38. $f(x) = \sqrt{x}\ln x \Rightarrow f'(x) = \frac{1}{2\sqrt{x}}\ln x + \frac{\sqrt{x}}{x} = \frac{\sqrt{x}}{x}(\frac{\ln x}{2}+1) = 0 \Rightarrow \ln x = -2 \Rightarrow x = e^{-2}$

$f(e^{-2}) = \sqrt{e^{-2}}\cdot \ln e^{-2} = -2e^{-1}$

39. $\iint_Q e^{-x^2-y^2}dxdy = \int_0^a\int_0^{2\pi} e^{-r^2}\cdot rd\theta dr = 2\pi\int_0^a e^{-r^2}\cdot rdr = 2\pi\left[-\frac{1}{2}e^{-r^2}\right]_0^a = -\pi\cdot e^{-a^2}+\pi = \pi(-e^{-a^2}+1)$

40. $\pi\int_0^1 x^2-(x^2)^2 dx = \pi\left[\frac{1}{3}x^3-\frac{1}{5}x^5\right]_0^1 = \frac{2}{15}\pi$

41. $f(x) = x^3+ax^2+bx+c \Rightarrow f'(x) = 3x^2+2ax+b = 3(x+1)(x-3) \Rightarrow a=-3, b=-9$

$\therefore f(-1) = (-1)^3-3(-1)^2-9(-1)+c = 7 \Rightarrow c = 2$

42. $f(x) = (\frac{x^2+1}{2})^9$， $f'(x) = 9x(\frac{x^2+1}{2})^8$, $f''(x) = 9(\frac{x^2+1}{2})^8 + 72x^2(\frac{x^2+1}{2})^7$

$\therefore f'(1)+f''(1)-(f'(1))^2 = 9+(9+72)-9^2 = 9$

43. $\int_{-2}^{2}(4-x^2)^{\frac{3}{2}}dx \overset{令 x=2\sin x}{=} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(4\cos^2 x)^{\frac{3}{2}}\cdot 2\cos xdx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}16\cos^4 xdx$

$= 16(\left[\frac{\cos^3 x\sin x}{4}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{3}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2 xdx) = 16\times\frac{3}{4}\times\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}dx = 6\pi$

44.概念

45.(A)$\langle g,g\rangle = \int_0^1(e^t\times e^t)dt = \left[\frac{e^{2t}}{2}\right]_0^1 = \frac{e^2-1}{2} \Rightarrow \|g\| = \sqrt{\langle g,g\rangle} = \sqrt{\frac{e^2-1}{2}}$

(B)$\langle f,f\rangle = \int_0^1(t\times t)dt = \left[\frac{1}{3}t^3\right]_0^1 = \frac{1}{3} \Rightarrow \|f\| = \sqrt{\langle f,f\rangle} = \sqrt{\frac{1}{3}}$

(C)$\langle f,g\rangle = \int_0^1(t\times e^t)dt = \left[(t-1)e^t\right]_0^1 = 1$

故選(D)

46. $T(z,w)=(2z+iw,(1-i)z)\Rightarrow [T]=\begin{bmatrix}2 & i\\ 1-i & 0\end{bmatrix}\Rightarrow [T^*]=[T]^*=\begin{bmatrix}2 & 1+i\\ -i & 0\end{bmatrix}$

$\therefore T^*(3-i,1+2i)=\begin{bmatrix}2 & 1+i\\ -i & 0\end{bmatrix}\begin{bmatrix}3-i\\ 1+2i\end{bmatrix}=\begin{bmatrix}5+i\\ -1-3i\end{bmatrix}$

47. $A=\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}\Rightarrow A^2=\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}\Rightarrow A^3=I$

(A) $A^{100}=(A^3)^{33}\cdot A=A$ (B) $A^{2012}=(A^3)^{670}\cdot A^2=A^2$ (C) $A^{99}=(A^3)^{33}=I$ (D) $\lim\limits_{n\to\infty}A^n$ 不存在

選(B)

48. $\begin{cases}\begin{pmatrix}a&b\\c&a\end{pmatrix}=a\begin{pmatrix}1&0\\0&1\end{pmatrix}+b\begin{pmatrix}0&1\\0&0\end{pmatrix}+c\begin{pmatrix}0&0\\1&0\end{pmatrix}\\ \begin{pmatrix}0&a\\-a&b\end{pmatrix}=a\begin{pmatrix}0&1\\-1&0\end{pmatrix}+b\begin{pmatrix}0&0\\0&1\end{pmatrix}\end{cases}\Rightarrow\begin{cases}\dim(w_1)=3\\ \dim(w_2)=2\end{cases}$

$\therefore \dim(w_1\cap w_2)+\dim(w_1+w_2)=\dim(w_1\cap w_2)+\dim(w_1)+\dim(w_2)-\dim(w_1\cap w_2)$

$=\dim(w_1)+\dim(w_2)=3+2=5$

49. $\because a<b<c\Rightarrow a+a+a=3a<a+b+c=12\Rightarrow a<4$

$\therefore (a,b,c)$ 可能爲 $(3,4,5)$ 、 $(2,3,7)$ 、 $(2,4,6)$ 、 $(1,2,9)$ 、 $(1,3,8)$ 、 $(1,4,7)$ 、 $(1,5,6)$ 共 7 種

所求 $\dfrac{7}{H^3_{12-3}}=\dfrac{7}{C^{11}_9}=\dfrac{7}{55}$

50.因爲「不大於 77 的正整數中與 77 互質」的有 $77\times(1-\dfrac{1}{7})\times(1-\dfrac{1}{11})=60$ 個，

由費馬小定理知 $2^{60}\equiv 1(\mathrm{mod}\,77)$ ，則

$2^{100000}=(2^{60})^{1666}\cdot 2^{40}\equiv 2^{40}=(2^{10})^4\equiv 23^4=529^2\equiv(-10)^2\equiv 23(\mathrm{mod}\,77)$