

1.開口向下之二次函數且對稱軸 $x=3$ 有最大值，故選(3)

2.實係數方程，共軛虛根知 $x^2 - 4x + 5$ 為其因式，所以

$$x^3 - ax^2 - bx - 10 = (x^2 - 4x + 5)(x - 2) = x^3 - 6x^2 + 13x - 10 = 0$$

$$3.(1) \det(cA) = c^n \cdot \det(A) \quad (2) \det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1 \neq \det \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \det \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = 0 \quad (3) \det(A^T) = \det(A)$$

故選(4)

$$4. \pi \int_1^4 (\sqrt{x} - 1)^2 dx = \pi \left[\frac{1}{2} x^2 - \frac{4}{3} x^{\frac{3}{2}} + x \right]_1^4 = \frac{7}{6} \pi$$

5.乙：三場內獲勝+四場內獲勝+5場內獲勝

$$P = \left(\frac{2}{5}\right)^3 + C_1^3 \cdot \frac{3}{5} \cdot \left(\frac{2}{5}\right)^3 + C_2^4 \cdot \left(\frac{3}{5}\right)^2 \cdot \left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)^3 \cdot \left(1 + \frac{9}{5} + \frac{54}{25}\right) = \frac{8}{125} \cdot \frac{124}{25} = \frac{992}{3125}$$

$$6. \overrightarrow{AH} = \overrightarrow{AO} + \overrightarrow{OH} = -\overrightarrow{OA} + \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{OC}$$

$\therefore \overrightarrow{OB} = \overrightarrow{OC} \therefore \overrightarrow{AH} \perp \overrightarrow{BC}$ ，同理 $\overrightarrow{BH} \perp \overrightarrow{AC}$ ， $\overrightarrow{CH} \perp \overrightarrow{AB}$ ， H 為三高交點，故選(4)

$$7. \angle ACB = 90^\circ \quad \overline{CD} = 2\sqrt{25^2 - \left(\frac{40 \times 30}{50}\right)^2} = 14，故選(4)$$

$$8. \frac{\Delta_x}{\Delta} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = 2, \quad \frac{\Delta_y}{\Delta} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = -3$$

$$\Rightarrow (x, y) = \left(\frac{\Delta_x}{\Delta}, \frac{\Delta_y}{\Delta}\right) = \left(\frac{\begin{vmatrix} 6c_1 & -2a_1 \\ 6c_2 & -2a_2 \end{vmatrix}}{\begin{vmatrix} 3b_1 & -2a_1 \\ 3b_2 & -2a_2 \end{vmatrix}}, \frac{\begin{vmatrix} 3b_1 & 6c_1 \\ 3b_2 & 6c_2 \end{vmatrix}}{\begin{vmatrix} 3b_1 & -2a_1 \\ 3b_2 & -2a_2 \end{vmatrix}}\right) = \left(\frac{12}{6} \cdot \frac{\Delta_y}{\Delta}, \frac{-18}{6} \cdot \frac{\Delta_x}{\Delta}\right) = (-6, -6)$$

$$9. \Delta = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & -2 \\ 1 & a & b \end{vmatrix} = -a + b + 5, \quad \Delta_x = \begin{vmatrix} 1 & 2 & -3 \\ -1 & 3 & -2 \\ 1 & a & b \end{vmatrix} = 5a + 5b + 5, \quad \Delta_y = \begin{vmatrix} 1 & 1 & -3 \\ 1 & -1 & -2 \\ 1 & 1 & b \end{vmatrix} = -2b - 6$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & -1 \\ 1 & a & 1 \end{vmatrix} = 2a - 4$$

(1) $\Delta \neq 0$, 恰有一組解 (2) $\Delta = 0$, $\Delta_x \neq 0$, 無解 (3) $\Delta = 0$, $\Delta_x \neq 0$, 無解

(4) $\Delta = 0$, $\Delta_x = \Delta_y = \Delta_z = 0$, 無限多解，故選(3)

$$10. \because \overline{PF_1} \text{ 中點在短軸上}, \therefore \text{由相似性質 } \overline{PF_2} \text{ 爲正焦弦的一半, 所求 } \frac{1}{2} \cdot \frac{2b^2}{a} = \frac{2(\sqrt{5})^2}{2 \times 5} = 1$$

$$11. a = \sqrt{2^2 + 3^2} = \sqrt{13}, \text{ 故選(2)}$$

$$12. (1) \left(\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} \right) \cdot (2, 3, 1) = -2 \quad (2) \left(\begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} \right) \cdot (2, 3, 1) = -4$$

$$(3) (-1, 1, -1) \cdot (2, 3, 1) = 0 \quad (4) (2, 1, 1) \cdot (2, 3, 1) = 8$$

故選(3)

$$13. 50 \times \frac{1}{6} = 8.\bar{3}, \text{ 由大數法則選(1)}$$

$$14. \frac{1}{\sqrt{k+1}} < \frac{1}{\sqrt{k}} < \frac{1}{\sqrt{k-1}} \Rightarrow \frac{1}{\sqrt{k} + \sqrt{k+1}} < \frac{1}{2\sqrt{k}} < \frac{1}{\sqrt{k} + \sqrt{k-1}}$$

$$\Rightarrow 2(\sqrt{k+1} - \sqrt{k}) < \frac{1}{\sqrt{k}} < 2(\sqrt{k} - \sqrt{k-1}) \Rightarrow \sum 2(\sqrt{k+1} - \sqrt{k}) < \sum \frac{1}{\sqrt{k}} < \sum 2(\sqrt{k} - \sqrt{k-1})$$

$$\Rightarrow 198 < 2(\sqrt{n+1} - 1) = 2(\sqrt{10001} - 1) < \sum \frac{1}{\sqrt{k}} < 2\sqrt{n} = 2\sqrt{10000} = 200$$

故選(2)

$$15. 3^{4 \times 3} = 3^{12}$$

16. 相鄰兩數間差值固定(1)10 (2)10 (3)9 (4)11, 可知(3)標準差最小, 故選(3)

$$17. (1) 2\sin(60^\circ + 74^\circ) = 2\sin 134^\circ \quad (2) 2\sin(60^\circ + 64^\circ) = 2\sin 124^\circ$$

$$(3) 2\sin(60^\circ + 54^\circ) = 2\sin 114^\circ \quad (4) 2\sin(60^\circ + 44^\circ) = 2\sin 104^\circ$$

故選(1)

$$18. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec x - \sqrt{2}}{x - \frac{\pi}{4}} = \lim_{x \rightarrow \frac{\pi}{4}} \sec x \tan x = \sqrt{2} \cdot 1 = \sqrt{2}$$

$$19. \lim_{\substack{x \rightarrow \infty \\ y \rightarrow 2}} \left(1 + \frac{1}{x}\right)^{\frac{x^2}{x+y}} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{\frac{x^2}{x}} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$20. \varepsilon^3 = 1 \text{ 且 } 1 + \varepsilon + \varepsilon^2 = 0$$

$$\begin{vmatrix} 1 & 1 & \varepsilon \\ 1 & 1 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{vmatrix} = 1 + \varepsilon^4 + \varepsilon^2 - \varepsilon^3 - \varepsilon^3 - 1 = 1 + \varepsilon + \varepsilon^2 - 1 - 1 - 1 = -3$$

$$21. \log 875^{16} = 16(3\log 5 + \log 7) = 16(3 \times 0.699 + 0.8451) = 47.0736, \text{ 有 } 47+1=48 \text{ 位數}$$

22.(1)Ratio test $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{100^{n+1}}{(n+1)!} \cdot \frac{n!}{100^n} \right| = \lim_{n \rightarrow \infty} \frac{100}{n+1} = 0$, 收斂

(2)Ratio test $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{[(n+1)!]^2}{[2(n+1)]!} \cdot \frac{(2n)!}{(n!)^2} \right| = \lim_{n \rightarrow \infty} \frac{(n+1)^2}{(2n+1)(2n+2)} = \lim_{n \rightarrow \infty} \frac{n+1}{2(2n+1)} = \frac{1}{4}$, 收斂

(3)Ratio test $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} \right| = \lim_{n \rightarrow \infty} \frac{n^n}{(n+1)^n} = \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n = \lim_{n \rightarrow \infty} \left(\frac{1}{1+\frac{1}{n}} \right)^n = e^{-1}$, 收斂

(4) $\sum_{n=1}^{\infty} \frac{1}{3n} = \sum_{n=1}^{\infty} \left(\frac{1}{3n} + \frac{1}{3n} - \frac{1}{3n} \right) < \sum_{n=1}^{\infty} \left(\frac{1}{3n-2} + \frac{1}{3n-1} - \frac{1}{3n} \right)$ 又 $\sum_{n=1}^{\infty} \frac{1}{3n} = \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n}$ 發散 ,

所以 $\sum_{n=1}^{\infty} \left(\frac{1}{3n-2} + \frac{1}{3n-1} - \frac{1}{3n} \right)$ 發散 , 故選(4)

23. $\because 11^{16} \equiv 1 \pmod{17} \therefore 11^{104} + 1 = (11^{16})^6 \cdot 11^8 + 1 \equiv (-6)^8 + 1 = (34+2)^4 + 1 \equiv 16+1 \equiv 0$

24. $9 : (09, 81, 29, 61, 49, 41, 69, 21, 89, 01)$, 則 $9^{9^9} = 9^{100k+89} \equiv 9^{89} \equiv 9^9 \equiv 89 \pmod{100}$

25.(1)① $x=1$ 代入 $1^3 + 6 \times 1^2 + 3 \times 1 + 3 \neq 0$ ② $x=-1$ 代入 $(-1)^3 + 6 \times (-1)^2 + 3 \times (-1) + 3 \neq 0$

③ $x=3$ 代入 $3^3 + 6 \times 3^2 + 3 \times 3 + 3 \neq 0$ ④ $x=-3$ 代入 $(-3)^3 + 6 \times (-3)^2 + 3 \times (-3) + 3 \neq 0$

(2)① $x=1$ 代入 $1^5 - 5 \times 1^3 + 15 \neq 0$ ② $x=-1$ 代入 $(-1)^5 - 5 \times (-1)^3 + 15 \neq 0$

③ $x=3$ 代入 $3^5 - 5 \times 3^3 + 15 \neq 0$ ④ $x=-3$ 代入 $(-3)^5 - 5 \times (-3)^3 + 15 \neq 0$

⑤ $x=5$ 代入 $5^5 - 5 \times 5^3 + 15 \neq 0$ ⑥ $x=-5$ 代入 $(-5)^5 - 5 \times (-5)^3 + 15 \neq 0$

⑦ $x=15$ 代入 $15^5 - 5 \times 15^3 + 15 \neq 0$ ⑧ $x=-15$ 代入 $(-15)^5 - 5 \times (-15)^3 + 15 \neq 0$

(3)① $x=1$ 代入 $1^4 + 1^3 + 1^2 + 1 + 1 \neq 0$ ② $x=-1$ 代入 $(-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 \neq 0$

有三個 , 故選(4)

26. 5、6 或 8 的倍數

$$\left\lceil \frac{1000}{5} \right\rceil + \left\lceil \frac{1000}{6} \right\rceil + \left\lceil \frac{1000}{8} \right\rceil - \left\lceil \frac{1000}{30} \right\rceil - \left\lceil \frac{1000}{24} \right\rceil - \left\lceil \frac{1000}{40} \right\rceil + \left\lceil \frac{1000}{120} \right\rceil$$

$$= 200 + 166 + 125 - 33 - 41 - 25 + 8 = 400$$

所求 $1000 - 400 = 600$

27. $C_2^4 = 6$

28. $H_6^3 = C_6^8 = 28$

29. $C_0^5 + C_2^5 + C_4^5 = 1 + 10 + 5 = 16$

30. $3^4 = 81$

31. $x = y + \ln y \Rightarrow 1 = y' + \frac{y'}{y} \Rightarrow y' = \frac{y}{1+y}$, 故選(3)

$$32. y^2 + 2\ln y = x^4 \Rightarrow 2ydy + \frac{2dy}{y} = 4x^3 dx \Rightarrow \frac{dy}{dx} = 4x^3 \cdot \frac{y}{2y^2 + 2} = \frac{2x^3 y}{y^2 + 1}$$

$$33. 10^{10} = 2^{10} \cdot 5^{10}, \text{ 所求 } (10+1)(10+1)=121$$

$$34. n^2 - 2n = \pm 1, \pm 2, \pm 3, \pm 6 \text{ 有整數解, 則 } n^2 - 2n = -1 \Rightarrow n = 1 \text{ 或 } n^2 - 2n = 3 \Rightarrow n = 3, -1$$

$$\text{所求 } 1 + 3 + (-1) = 3$$

$$35. 50 \times 4.2 = 210, \text{ 中位數爲 } a_{25}、a_{26} \text{ 的平均值}$$

$$(4) 26 \times 7.5 = 195 \Rightarrow 210 - 195 = 15 < 24 \text{ (不合)} \quad (3) 26 \times 7 = 182 \Rightarrow 210 - 182 = 28 > 24$$

$$\text{故選(3)}$$

$$36. f(x) = \left| \log_{\frac{1}{2}} x \right| \Rightarrow x \in \left[\frac{1}{2}, 4 \right], f(x) \in [-2, 1] \Rightarrow f(x) \in [0, 2], \text{ 所求 } 2 - 0 = 2$$

$$37. \lim_{n \rightarrow \infty} \frac{1}{\sqrt[n]{|a_n|}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[n]{\left| \frac{1}{2^n} \right|}} = \lim_{n \rightarrow \infty} 2 = 2 \Rightarrow |x^n - 0| < 2 \Rightarrow |x| < \sqrt[n]{2} = 1, \text{ 故選(4)}$$

$$38. \int e^x \sin x dx = \sin x e^x - \int e^x \cos x dx = \sin x e^x - \cos x e^x - \int e^x \sin x dx$$

$$\Rightarrow \int_0^\pi e^x \sin x dx = \frac{1}{2} [\sin x e^x - \cos x e^x]_0^\pi = \frac{1}{2} (e^\pi + 1)$$

$$39. \text{橢圓定義, 到兩定點距離和爲定值, 故選(3)}$$

$$40. \text{分爲兩對皆分別同組及僅有一對在同組}$$

$$\frac{C_2^{16} \cdot C_2^{14} \cdot C_2^{12} \cdot C_2^{10} \cdot C_2^8 \cdot C_2^6 \cdot C_2^4 \cdot C_2^2}{8!} + \frac{C_1^2 \cdot 16 \cdot 15 \cdot C_2^{14} \cdot C_2^{12} \cdot C_2^{10} \cdot C_2^8 \cdot C_2^6 \cdot C_2^4 \cdot C_2^2}{7!}$$

$$\frac{C_2^{20} \cdot C_2^{18} \cdot C_2^{16} \cdot C_2^{14} \cdot C_2^{12} \cdot C_2^{10} \cdot C_2^8 \cdot C_2^6 \cdot C_2^4 \cdot C_2^2}{10!}$$

$$= \frac{\frac{C_2^{16}}{8} + \frac{2 \times 16 \times 15}{1}}{\frac{C_2^{20} \cdot C_2^{18} \cdot C_2^{16}}{10 \times 9 \times 8}} = \frac{15 + 2 \times 16 \times 15}{19 \times 17 \times 15} = \frac{1 + 2 \times 16}{19 \times 17} = \frac{33}{19 \times 17}$$

$$41. \frac{1}{2} = \frac{2^2 + \overline{BC}^2 - 3^2}{2 \cdot 2 \cdot \overline{BC}} \Rightarrow \overline{BC} = 1 + \sqrt{6}, \text{ 所以 } \Delta ABC = \frac{1}{2} \cdot 2 \cdot (1 + \sqrt{6}) \cdot \sin 60^\circ = \frac{\sqrt{3} + \sqrt{18}}{2}$$

$$42. \text{令 } f(x) = (x^2 + 1)(ax + b) + 2x - 1 \text{ 將 } x = \frac{1}{2} \text{ 代入 } \frac{5}{4} \left(\frac{a}{2} + b \right) + 1 - 1 = 5 \Rightarrow \frac{a}{2} + b = 4 \Rightarrow a + 2b = 8$$

$$43. A \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \cdot \frac{1}{-2} \cdot \begin{bmatrix} -1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -2 \\ \frac{3}{2} & -1 \end{bmatrix}$$

$$\text{所求 } x = A^{-1}b = \frac{2}{5} \begin{bmatrix} -1 & 2 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \\ \frac{5}{5} \\ -\frac{2}{5} \end{bmatrix}$$

44. $f(x)$ 為連續函數且在閉區間，所以有最大值及最小值存在 \rightarrow (A)、(B) 正確
(C) 不一定 (D) 不一定，故選 (4)

45. 兩邊平方 $x-5 = m^2 x_2 + 4mx + 4 \Rightarrow (4m-1)^2 - 4m^2 \cdot 9 > 0 \Rightarrow (10m-1)(-2m-1) > 0$
 $-0.5 < m < 0.1$ 繪圖可知 $m > 0 \therefore 0 < m < 0.1$ ，故選 (1)

$$46. \cos \theta = \frac{(1,2,0) \cdot (3,2,6)}{\sqrt{5} \cdot \sqrt{49}} = \frac{7}{7\sqrt{5}} = \frac{\sqrt{5}}{5}$$

47. ① $n = 5k + 3 \Rightarrow -10 \sim 9$ 共 20 個 ② $n = 3m + 1 \Rightarrow -17 \sim 16$ 共 34 個

③ $n = 15t - 2 \Rightarrow -3 \sim 3$ 共 7 個 (被 3 除餘 1 且被 5 除餘 3)

所求 $20 + 34 - 7 = 47$

48. (1) 反例：折線，故選 (1)

49. $(1, -1)$ 、 $(3, 0)$ 、 $(3, 2)$ 、 $(1, 1)$ 為平行四邊形，所求 $(2-0) \times (3-1) = 4$

50. $x^2 + y^2 - 2x + 4y = 0 \Rightarrow (x-1)^2 + (y+2)^2 = 5$ 且 $\sqrt{(5-1)^2 + (1-(-2))^2} = 5$

$$\therefore 2 \cdot \frac{1}{2} \cdot \sqrt{5} \cdot 2\sqrt{5} = \frac{1}{2} \cdot 5 \cdot \overline{AB} \Rightarrow \overline{AB} = 4$$

