

1.

$$\overrightarrow{OP} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \sin \alpha \\ \cos \beta \end{bmatrix},$$

$$\text{又} \begin{cases} 0 \leq \alpha \leq \frac{\pi}{3} \\ 0 \leq \beta \leq \frac{\pi}{6} \end{cases} \Rightarrow \begin{cases} 0 \leq \sin \alpha \leq \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} \leq \cos \beta \leq 1 \end{cases}$$

$$\text{所以 面積} = \left(\frac{\sqrt{3}}{2} - 0\right) \cdot \left(1 - \frac{\sqrt{3}}{2}\right) \cdot \det \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = 4\sqrt{3} - 6$$