

8.  $29^{202}$  除以 13 的餘數是多少？

- (A) 1 (B) 3 (C) 9 (D) 11

解： $29^{202} = (26+3)^{202} \equiv 3^{202} = (3^3)^{67} 3 \equiv 3 \pmod{13}$

20. 已知實數  $x, y$  滿足條件  $\sin x + \sin y = \frac{\sqrt{2}}{2}$  與  $\cos x + \cos y = \frac{\sqrt{6}}{2}$ ，則  $\sin(x+y)$  之值為何？

- (A) 0 (B)  $\frac{\sqrt{2}}{2}$  (C)  $\frac{\sqrt{3}}{2}$  (D) 1

解：兩式分別和差化積，

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

相除後可得  $\tan\left(\frac{x+y}{2}\right) = \frac{1}{\sqrt{3}}$

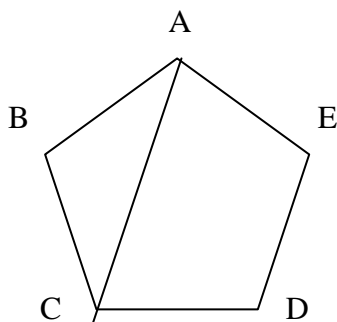
$$\therefore \frac{x+y}{2} = \frac{\pi}{6} + k\pi$$

$$\Rightarrow x+y = \frac{\pi}{3} + 2k\pi$$

$$\sin(x+y) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

34. 設正五邊形的邊長為 2，則其對角線長為多少？

- (A)  $\sqrt{5}+1$  (B)  $2\sqrt{5}-1$  (C)  $3+\sqrt{5}$  (D)  $2\sqrt{5}-3$



$$\begin{aligned}\overline{AC}^2 &= 2^2 + 2^2 - 2 \times 2 \times 2 \times \cos 108^\circ \\ &= 8 + 8 \sin 18^\circ = 8 + 8 \times \frac{\sqrt{5}-1}{4} \\ &= 6 + 2\sqrt{5}\end{aligned}$$

$$\overline{AC} = \sqrt{6+2\sqrt{5}} = \sqrt{(\sqrt{5}+1)^2} = \sqrt{5}+1$$